Simple approximation for the starting-energy-independent two-body effective interaction with applications to ^{6}Li

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Abstract

We apply the Lee-Suzuki iteration method to calculate the linked-folded diagram series for a new Nijmegen local NN potential. We obtain an exact starting-energy-independent effective two-body interaction for a multi-shell, no-core, harmonic-oscillator model space. It is found that the resulting effective-interaction matrix elements can be well approximated by the Brueckner G-matrix elements evaluated at starting energies selected in a simple way. These starting energies are closely related to the energies of the initial two-particle states in the ladder diagrams. The "exact" and approximate effective interactions are used to calculate the energy spectrum of ^{6}Li in order to test the utility of the approximate form.

1 Introduction

Conventional shell-model (SM) calculations often assume an inert core with a few valence nucleons as active particles. The calculation of the two-body effective interaction for the valence nucleons can be conveniently divided into three steps: (1) Calculate the Brueckner reaction matrix G [1] from a realistic NN potential; (2) Calculate the two-body Q-box [2] from the G-matrix; and (3) Calculate the folded diagrams [3] from the Q-box. The second step provides major difficulties, because one is unable to evaluate the core-polarization diagrams to all orders and there is no sign of convergence within the lowest few orders [4]. This difficulty might be avoided by the use of a no-core model space [5], for which all the nucleons in a nucleus are treated as active. In such a model space, because there are no hole lines, all the core-polarization diagrams are absent and the two-body Q-box reduces to the G-matrix. The folded diagrams can be calculated from the Q-box by using iteration methods proposed by Kuo and Krenciglowa [6] or by Lee and Suzuki [7]. Beyond these issues are the largely unexplored questions on the role of effective many-body forces in no-core model spaces.

In previous works [8, 9], we calculated the low-lying energy spectra for a few light nuclei by employing no-core model spaces. We approximated the two-body effective interaction by the G-matrix and neglected the folded diagrams. Because of this approximation, our calculations involved the starting energy as a parameter, which, in one study [9], was chosen to fit the nuclear binding energy.

In this work, we will calculate the G-matrix using an improved version of the Nijmegen potential (NijmII) [10] and we follow the Lee and Suzuki method [7] to sum the two-body folded diagrams to all orders. The resulting starting-energy-independent two-body effective interaction is not Hermitian but its non-Hermiticity is found to be extremely small. We obtain the Hermitian effective interaction $v_{\rm eff}^{(2)}$ to be used in SM calculation by taking the average of the non-Hermitian effective interaction and its conjugate. It is pointed out in Ref.[11] that this is an excellent approximation.

We will also discuss the choice of the two-nucleon Hamiltonian $H^{(2)}$

employed in the G-matrix calculation, which determines the intermediate-energy spectrum in the ladder diagrams. Currently, due to uncertainties in the optimal choices of one-body potentials and methods for treating the spurious center-of-mass motion, there is no generally accepted $H^{(2)}$. It is obvious that the two-body effective interactions $v_{\rm eff}^{(2)}$ depend on $H^{(2)}$, so that one wishes to employ the $H^{(2)}$ which best represents the physics of the two-nucleon subsystem in the nuclear medium. We will present a choice which is physically motivated yet retains simplicity for calculations.

We will furthermore introduce an approximation scheme which allows us to obtain easily, an effective two-body interaction for a no-core model space directly from the starting-energy-dependent G-matrix without evaluating the folded diagrams. We will demonstrate that the resulting approximate form is an improvement over the procedure we introduced in Ref.[9] and more closely represents the exact theoretical $v_{\rm eff}^{(2)}$.

2 G-matrix and Two-body Effective Interaction

Assuming that there are only two-body interactions among nucleons in a nucleus, the nuclear Hamiltonian for an A-nucleon nucleus can be written as:

$$H = \left(\sum_{i=1}^{A} t_i - T_{\rm CM} \right) + \sum_{i<j}^{A} v_{ij}, \tag{1}$$

where t is the one-body kinetic energy, $T_{\rm CM}$ is the center-of-mass (CM) kinetic energy of the nucleus, v is the two-body NN potential.

The expression for the Brueckner reaction matrix G [1] can be generally written in the following form:

$$G_{12}(\omega) = v_{12} + v_{12} \frac{Q}{\omega - H^{(2)}} v_{12}, \tag{2}$$

where Q is the Pauli projection operator and ω is the starting energy. The two-nucleon Hamiltonian $H^{(2)}$ represents the dynamics of the two-particle subsystem in the nuclear medium generated by the remaining (A-2) nucleons (also referred to as spectators). It makes sense to optimize the description

of this two-particle subsystem in order to minimize the effects of the many-body effective interactions which we plan to neglect. Clearly the role of the medium on the single-particle states needs to be included and one is, therefore, led to introduce one-body potentials into $H^{(2)}$ yielding:

$$H^{(2)} = (t_1 + t_2) + v_{12} + \mathcal{V}_1 + \mathcal{V}_2 \equiv (h_1 + h_2) + v_{12}, \tag{3}$$

where $h = t + \mathcal{V}$ is the single-particle (SP) Hamiltonian with t the one-body kinetic energy and $\mathcal{V}$ the mean field generated by the spectators.

A physically motivated $\mathcal{V}(\boldsymbol{r})$ could be the Hartree-Fock (HF) self-consistent mean field or the phenomenological Woods-Saxon (WS) well with appropriate depth, width and surface thickness. However, to employ either the HF field or the WS well is a very computationally demanding project for a realistic force v_{12}. For convenience, we assume that the mean field $\mathcal{V}$ can be approximated by a *shifted* harmonic-oscillator (HO) potential (see Fig.1), namely,

$$\mathcal{V}(\boldsymbol{r}) \simeq \mathcal{V}_{\rm HO}^{\rm shifted}(r) \equiv -V_0 + \frac{1}{2} m\Omega^2 r^2 = -V_0 + u^{\rm HO}(r). \tag{4}$$

The use of the HO potential in conjunction of the HO basis not only simplifies the G-matrix calculation [12] but also facilitates the treatment of the spurious center-of-mass motion.

Note that, as far as the low-lying states are concerned, one need not be greatly concerned with the obvious fact that, when r goes to infinity, $\mathcal{V}(\boldsymbol{r})$ vanishes while $u^{\rm HO}(r)$ becomes infinite. Actually, in the low-lying states, the nucleons in a nucleus remain primarily within the nuclear radius R_A, so the shape of $\mathcal{V}(r)$ for large r, say, $r > 2R_A$, plays a less significant role in the bound-state spectrum.

With the approximation stated in Eq.(4), the two-nucleon Hamiltonian (3) becomes

$$H^{(2)} \simeq (t_1+t_2)+v_{12}+(u_1^{\rm HO}-V_0)+(u_2^{\rm HO}-V_0) \equiv (h_1^{\rm HO}+h_2^{\rm HO})+v_{12}-2V_0, \tag{5}$$

where $h^{\rm HO} = t + u^{\rm HO}$ is the pure HO SP Hamiltonian. The corresponding

G-matrix (2) becomes

$$G_{12}(\omega) \simeq v_{12} + v_{12}\frac{Q}{\omega' - (h_1^{\rm HO} + h_2^{\rm HO} + v_{12})}v_{12} \equiv G_{12}^{\rm HO}(\omega'), \qquad (6)$$

where $\omega' \equiv \omega + 2V_0$. In writing the above equation, we have added a constant $(2V_0)$ to both the starting energy ω and the two-nucleon Hamiltonian $H^{(2)}$. Obviously this does not change the result for the G-matrix which depends only on the difference between ω and $H^{(2)}$.

For no-core model spaces, the starting-energy-independent two-body effective interaction $v_{\rm eff}^{(2)}$ is the ladder diagram series (G-matrix) plus the folded diagrams. If we approximate the G-matrix by $G^{\rm HO}(\omega')$ as in Eq.(6), $v_{\rm eff}^{(2)}$ is written as

$$v_{\rm eff}^{(2)} \simeq G^{\rm HO}(\omega') + (\text{Folded diagrams}). \qquad (7)$$

The folded diagrams can be evaluated by employing the iteration methods proposed in Refs.[6, 7]. We will use the "vertex-renormalization" procedure of Ref.[7]. The input to this method consists of the $G(\omega')$ and its derivatives with respective to ω', whose values are taken at an arbitrary (in principle) but fixed starting energy ω'.

We use a HO SP basis with $\hbar\Omega$=18 MeV and a no-core model space containing the first 4 major shells ($0s$, $0p$, $1s$-$0d$ and $1p$-$0f$). The Q operator in Eq.(6) is defined to forbid the scattering of the two particles into an intermediate state inside the model space (i.e., Q=0).

For v_{12}, we adopt a new Nijmegen local NN potential (NijmII) [10], which was fitted to the world NN scattering data with a nearly optimal χ^2 per degree of freedom (1.03 per datum). Other potentials (Reid93 and AV18) obtained by fitting the same data have a comparable χ^2 and yield similar deuteron and triton properties (see Ref.[13] for more details). The derivatives of the G-matrix are calculated numerically through ninth order using 11 sets of $G(\omega')$ with ω' ranging from -75 MeV to +75 MeV, in steps of 15 MeV. In Table 1 under column "$v_{\rm eff}^{(2)}$", we list a few diagonal two-body matrix elements (TBMEs) of the resulting effective interaction. For comparison and further discussion below, we also present, in Table 1, G-

Table 1: Selected diagonal TBMEs for the starting-energy-independent two-body effective interaction $v_{\rm eff}^{(2)}$. These matrix elements should be compared with the G-matrix elements listed in the table for three different starting energies. Values in italics indicate values in good agreement with $v_{\rm eff}^{(2)}$. Note that $2\epsilon_{0s}^{\rm HO} = 3\hbar\Omega = 54$ MeV, $2\epsilon_{0p}^{\rm HO} = 5\hbar\Omega = 90$ MeV, $2\epsilon_{0d}^{\rm HO} = 7\hbar\Omega = 126$ MeV.

State	$v_{\rm eff}^{(2)}$	$G(\omega' = 30)$	$G(\omega' = 75)$	$G(\omega' = 110)$	"Approx."
$(0s_{1/2}^2)^{J=0,T=1}$	-8.75	*-8.73*	-9.05	-9.47	-8.75
$(0s_{1/2}^2)^{J=1,T=0}$	-11.78	*-11.70*	-14.18	-20.64	-11.83
$(0p_{3/2}^2)^{J=0,T=1}$	-3.87	-3.54	*-3.86*	-4.35	-3.81
$(0p_{3/2}^2)^{J=1,T=0}$	-2.28	-1.33	*-2.34*	-4.23	-2.17
$(0d_{5/2}^2)^{J=0,T=1}$	-1.79	-1.09	-1.38	*-1.68*	-1.62
$(0d_{5/2}^2)^{J=1,T=0}$	-0.78	0.76	0.05	*-0.84*	-0.65

matrix elements at selected values of ω', and at a state-dependent choice of ω'.

3 Approximation to $v_{\rm eff}^{(2)}$

As shown in Eq.(7), for a *no-core* model space, the ω-independent $v_{\rm eff}^{(2)}$ is the sum of the ladder diagrams and the folded diagrams, both of which depend separately on the starting energy ω (or ω'). When the folded diagrams are ignored, as was often done in the past, one approximates $v_{\rm eff}^{(2)}$ by $G(\omega')$. It is obvious that the contribution of the folded diagrams correlates with ω. Below, we show that a particular choice of ω minimizes the root-mean-square (rms) contributions of the folded diagrams to the TBMEs of $v_{\rm eff}^{(2)}$. We further show that a state-dependent choice of ω' yields a remarkably good approximation to $v_{\rm eff}^{(2)}$.

In the full theory of the effective Hamiltonian, one has, in principle, independence of the mean field $\mathcal{V}$ and of ω. However, in practical calculations, to minimize the need to calculate higher-order processes, one wishes to make

physically sensible choices for these quantities such as discussed above in the case of $\mathcal{V}$.

We now consider arguments that may be presented to suggest a phenomenological choice for ω that could also simplify the calculation of the effective two-body interaction. It is generally accepted that the starting energy ω represents the initial energy E_2 of the two nucleons in the nuclear medium. In G-matrix calculations, the two nucleons are treated as two interacting particles moving in the mean field $\mathcal{V}$. We can think of the energy E_2 for a two-particle state, which is predominantly $|ab\rangle_{J,T}$ (a and b are the HO SP orbitals), as given by:

$$E_2 = \epsilon_a + \epsilon_b + \Delta, \tag{8}$$

where ϵ_a and ϵ_b are eigenenergies of the one-body Hamiltonian $(t + \mathcal{V})$. The quantity Δ represents the interaction energy and depends implicitly on the two-particle state $|ab\rangle_{J,T}$.

When the mean field $\mathcal{V}$ is approximated by the shifted HO potential as we did in Eq.(4), Eq.(8) becomes:

$$E_2 \simeq (\epsilon_a^{\rm HO} - V_0) + (\epsilon_b^{\rm HO} - V_0) + \Delta, \tag{9}$$

where $\epsilon_a^{\rm HO}$ and $\epsilon_b^{\rm HO}$ are the HO SP energies [$\epsilon_i^{\rm HO} = (2n_i + l_i + \frac{3}{2})\hbar\Omega$ with $i = a, b$].

Since the shifted starting energy ω' used in Eq.(6) for the G-matrix is related to the original starting energy ω through $\omega' = \omega + 2V_0$, we have the following equation for ω':

$$\omega' \simeq \epsilon_a^{\rm HO} + \epsilon_b^{\rm HO} + \Delta. \tag{10}$$

Note that although E_2 and, thus, ω are negative for a two-particle state bound in the nucleus, ω' is not necessarily negative, because the quantity V_0 (approximately representing the depth of the potential well generated by the spectators) is always positive. This provides a partial justification to the choice of ω' made in Ref.[9], where G-matrices at a positive ω' are found to yield the approximately correct nuclear binding energies.

We, therefore, expect the G-matrix for the starting energy ω' given by Eq.(10) to be a reasonable approximation to $v_{\rm eff}^{(2)}$. This is clearly demonstrated in Table 1, where the TBMEs of $G(\omega')$ for three values of ω' are listed. From Table 1, one can see that the listed TBMEs of $v_{\rm eff}^{(2)}$ can be approximated by those of $G^{\rm HO}(\omega')$ at a starting energy ω' given by Eq.(10) with Δ ranging from –25 MeV to –15 MeV (numbers in italic). As we mentioned before, Δ more or less represents the state-dependent contribution to the two-nucleon energy from the NN interaction.

If one picks Δ for each matrix element such that $G(\omega') = V_{\rm eff}^{(2)}$, then from Table 1, one can also see that Δ is generally larger in magnitude for the J=1, T=0 channel than for the J=0, T=1 channel. This is physically sensible since the NN interaction for the former channel is more attractive. It is also obvious from the table that Δ is larger in magnitude for lower-lying two-nucleon states (e.g., $0s_{1/2}^2$) than for higher-lying ones (e.g., $0p_{3/2}^2$ or $0d_{5/2}^2$), which accounts for the fact that the NN interaction is stronger for the former states.

The observations above lead us to suggest a simple state-dependent choice for ω'. That is, we suggest using Eq.(10) with Δ taken as a single constant for all states for simplicity. In this way, we hope that $G(\omega')$ will become a good approximation to $v_{\rm eff}^{(2)}$. We see that this is, indeed, the case by comparing the "exact" results in the first column of Table 1 with the "approximate" results [i.e. those obtained with ω' of Eq.(10) using $\Delta = -21$MeV] in the last column of Table 1.

To further illustrate the difference between a state-independent and our state-dependent choice of ω', we define an rms deviation of the matrix elements of G from those of the starting-energy-independent $v_{\rm eff}^{(2)}$ as

$$\delta(\omega') = \left\{ \frac{1}{N} \sum \left[\langle ab|v_{\rm eff}^{(2)}|cd\rangle_{J,T} - \langle ab|G(\omega')|cd\rangle_{J,T} \right]^2 \right\}^{1/2}, \qquad (11)$$

where the summation runs over all the N=332 two-body matrix elements for the SM space consisting of the first *three* major shells. Note that our full model space contains the first four major shells but we are less concerned about the matrix elements involving the highest shell, so we omit them from

the definition of the rms deviation.

For a fixed starting energy ω', $\delta(\omega')$ is plotted in Fig.2 as a solid curve. From the figure, one sees that the smallest rms deviation of about 0.42 MeV is obtained when the starting energy ω'=75 MeV.

In Fig.2 we show as a straight dashed line, the rms deviation when the starting energy ω' is chosen according to the prescription in Eq.(10) with Δ=-21 MeV. With this prescription of the starting energy, the G-matrix approximates $v_{\rm eff}^{(2)}$ rather well since the rms deviation is only 0.13 MeV.

4 Applications to ^{6}Li

We now use the "exact" and "approximate" effective interactions $v_{\rm eff}^{(2)}$ to perform SM calculations for ^{6}Li. The SM effective Hamiltonian is written as

$$H_{\rm SM} = \left(\sum_{i=1} t_i - T_{\rm CM} \right) + \sum_{i<j}^{A} v_{\rm eff}^{(2)}(ij). \qquad (12)$$

The contribution of the center-of-mass spurious motion is removed by adding $\lambda(H_{\rm cm} - \frac{3}{2}\hbar\Omega)$ (with $\lambda \gg 1$) to the above Hamiltonian. This is a feature available with the OXBASH SM code [14].

When one compares Eq.(1) and the above equation, Eq.(12), one sees that $v_{\rm eff}^{(2)}(ij)$ is in the position of v_{ij}. Namely, we are replacing the free NN potential by the effective two-body interaction. Here we wish to point out that the SP potential ($u^{\rm HO}$) was used only to determine the intermediate-energy spectrum in calculating $v_{\rm eff}^{(2)}(ij)$ from v_{ij} and the SP wavefunctions of the basis space. In principle, one expects some contributions from higher-order SP insertions. We have not calculated them in the present investigations. In Ref.[15], it has been shown that higher-order SP insertions have a negligible effect in large no-core space SM calculations.

In Table 2, we show the results for the low-lying energy spectrum of ^{6}Li. The calculations are performed in the same model space for which the G-matrices and the effective interaction are calculated. But we only allow up to $4\hbar\Omega$ excitations from the lowest-energy configuration $[(0s)^4(0p)^2]$. With

Table 2: The calculated and experimental low-lying energy spectrum for ^{6}Li using the "exact" and "approximate" effective interactions as discussed in the text. The results obtained using $G^{\rm HO}(\omega')$ at a constant starting energy ($\omega' = 0.0$MeV and $\omega' = 75.0$MeV) are also listed. For the ground state, the absolute energy is given. For the excited states, the excitation energies are given. All energies are in units of MeV. Since we have not included the Coulomb interaction, the experimental ground-state energy shown in the table is Coulomb corrected: $-31.996 - E_{\rm Coulomb} = -33.996$, where $E_{\rm Coulomb} = 2.0$ MeV is obtained from a HF calculation with the Skyrme 3 interaction.

$J^\pi_n(T)$	$G(\omega'=0)$	$G(\omega'=75)$	"Approx."	"Exact"	Experiment
$1^+_1(0)$	-21.497	-48.386	-36.854	-35.655	-33.996
$3^+_1(0)$	2.481	2.200	1.916	2.054	2.186
$0^+_1(1)$	2.544	5.246	5.168	4.932	3.563
$2^+_1(0)$	4.955	6.638	6.161	6.306	4.31
$2^+_1(1)$	5.660	8.472	8.406	8.125	5.37
$1^+_2(0)$	7.514	10.057	9.438	9.336	5.65
$1^+_1(1)$	11.295	15.994	15.719	15.372	(N/A)

the HO SP basis that we used ($\hbar\Omega$=18 MeV), $v_{\rm eff}^{(2)}$ overbinds the ground state by about 1.66 MeV, as shown in the column labelled "Exact". It should be pointed out that this result depends quite sensitively on the HO parameter $\hbar\Omega$. Obviously, this is related to the approximation in Eq.(4), whose quality depends on $\hbar\Omega$. Anyway, we find that when a HO basis with a smaller $\hbar\Omega$ is used, the resulting two-body effective interaction tends to overbind ^{6}Li by an even larger amount. We notice that this is also a feature of the results obtained by Poppelier and Brussaard in Ref.[16] (see Fig.7 in this reference), although in that work, the effective interaction has some residual dependence on the starting energy.

The calculated excitation energies shown in Table 2 for $v_{\rm eff}^{(2)}$ tend to be higher than the experimental results but the experimental level sequence is more or less reproduced. In Ref.[16], the excitation energies are even higher.

It is not clear to us why the effective interactions obtained through the Lee-Suzuki procedure from the G-matrices using an HO SP basis with $\hbar\Omega$ smaller than 18 MeV tend to overbind ^{6}Li. It is quite likely that for light nuclei, the approximation made in Eq.(4) of replacing the mean field generated by the spectators by a shifted HO potential with $\hbar\Omega < 18$MeV requires significant corrections such as effective three-body forces. Further investigations on this are necessary.

Shown in Table 2 under column "Approx." are the results of the SM calculation using the G-matrix (instead of $v_{\rm eff}^{(2)}$) calculated at the starting energies given by Eq.(10) with $\Delta = -21$MeV. This G-matrix has been demonstrated in the previous section to be a good approximation to $v_{\rm eff}^{(2)}$ when the individual matrix elements are compared (see Table 1 and Fig.1). Apparently it is also a good approximation to $v_{\rm eff}^{(2)}$ when tested by evaluating the energy spectrum.

In contrast, the G-matrix evaluated at any constant starting energy ω' is not a very good approximation to $v_{\rm eff}^{(2)}$. One can see from Table 2 that the calculated ground-state binding energy of ^{6}Li using G-matrix at $\omega' = 0$ is about 14 MeV smaller than the "exact" result (-21.497 MeV vs -35.655 MeV). The calculated energy spectrum is also different. Indeed this G-

matrix has an rms deviation of about 1 MeV in its TBMEs from those of $v_{\rm eff}^{(2)}$ (see Fig.2) and should not be expected to approximate $v_{\rm eff}^{(2)}$ well.

When the starting energy ω' is restricted to be a constant, the G-matrix at $\omega' = 75$ best approximates $v_{\rm eff}^{(2)}$ when the rms deviation $\delta(\omega')$ in the TBMEs [Eq.(11)] is used as the criterion (Fig.2). The SM results using $G(\omega' = 75)$ are also listed in Table 2. The ground-state energy is clearly too low compared to the "exact" result.

5 Conclusions

We have succeeded in evaluating a starting-energy-independent effective two-body interaction $v_{\rm eff}^{(2)}$ for a large no-core model space for the new Nijmegen potential [10]. Our main conclusion is that $v_{\rm eff}^{(2)}$ can be well-approximated by the G-matrix elements evaluated at a starting energy which depends on the energy of the initial two-particle state in the ladder diagrams. We have seen from Table 1 and Fig.1 that for the effective-interaction TBME $\langle ab|v_{\rm eff}^{(2)}|cd\rangle_{J,T}$ compares favorably with the corresponding TBME of $G(\omega')$ where ω' is given by

$$\omega' = \epsilon_a^{\rm HO} + \epsilon_b^{\rm HO} + \Delta. \tag{13}$$

For ^{6}Li with $\hbar\Omega$=18MeV, we found that $\Delta = -21$MeV is a good choice. Notice that when ω' is away from the poles of $G(\omega')$, a variation of a few MeV in Δ is not significant as the G-matrix element is a very slowly varying function of ω' in this case.

We emphasize that the choice of $H^{(2)}$ could be very important, as it determines the intermediate-energy spectrum in the two-nucleon multiple scattering processes (ladder diagrams). Different $H^{(2)}$ will lead to different $v_{\rm eff}^{(2)}$. In this work, we have approximated the mean field generated by the spectator particles by a shifted HO potential, which seems to give a reasonable description for ^{6}Li, when the HO characteristic parameter $\hbar\Omega$=18 MeV is used.

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Figure Captions

Fig.1 The phenomenological, the shifted HO and the pure HO potential wells. The mean field $\mathcal{V}(r)$ generated by the $(A-2)$ spectator nucleons may be approximated by the phenomenological well, which is further approximated by the shifted HO potential for convenience. In the G-matrix calculation, we have added a constant shift $2V_0$ to both the two-nucleon Hamiltonian $H^{(2)}$ and the starting energy ω, so the pure HO potential is used along with a shifted starting energy $\omega' = \omega + 2V_0$. The same amount of shift added to $H^{(2)}$ and ω clearly does not have any effect on the G-matrix, because only the difference $\omega - H^{(2)}$ enters the G-matrix equation (6).

Fig.2 The rms deviation $\delta(\omega')$ of the two-body matrix elements of G at a fixed starting energy ω' from those of the starting-energy independent two-body effective interaction $v_{\rm eff}^{(2)}$, as defined by Eq(11) (solid curve). The horizontal dashed line is the rms deviation when the starting energy for the G-matrix elements is given by the state-dependent choice in Eq.(10) with $\Delta = -21$MeV.

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